“Complexity Law and Economics”
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Methodologies of Law and Economics (Thomas Ulen ed. 2014)

At the opening of the European Central Bank’s “Central Banking Conference in 2010, ECB President Jean-Claude Trichet reflected on the lessons from the financial crisis for macroeconomics and finance. “As a policy-maker during the crisis,” I found the available models of limited help,” he said. “In fact, I would go further: in the face of the crisis, we felt abandoned by conventional tools.”¹ Trichet made clear that he still found conventional economic tools like the dynamic stochastic general equilibrium and asset-pricing models quite useful. It was just that those more general economic frameworks needed supplementing in order to improve their robustness.

A number of economists have been pushing to develop a set of such complementary tools, and to develop a model of the economy that widens the lens of inquiry.² They are drawing from a line of argument that goes back hundreds of years, to Smith, Keynes, Schumpeter, and then more recently, to modern critics of neoclassical theory.

These scholars describe the economy as a complex adaptive system, one that continually changes and is always in the process of formation. A complex system is made up of elements that interact with each other to form complex patterns of regularity and novelty that unfold over time.³ In other fields, these elements might be cells in an immune system, neighbors in a community, ions in a spin glass, or ants in a colony. In economics, these elements might be traders in a market or homebuyers in a housing market. More generally, complexity economics investigates the dynamics of these interacting agents and the process of their evolution.

Of course, unlike cells in the immune system, economic actors in a market can consider in advance the outcomes of the choices they make, a feature of social

¹ Jean-Claude Trichet, Reflection on the nature of monetary policy non-standard measures and finance theory," Opening Address, ECB Central Banking Conference, November 18, 2010.

² For a good list of central scholarship in the field of complexity economics, see Herbert Gintis, The Economy as a Complex Adaptive System, 44 J. Econ. Lit. 4 (2006). For a review of agent-based computational economics, see Leigh Tesfatsion and Kenneth Judd, Handbook of Computational Economics II: Agent-Based Computational Economics (J. Holland and D. North eds. 2006).

systems that adds a layer of complication. But as behavioral economists have explained, human decision-making is boundedly rational and myopic, lacking the information or cognitive power to assess payoffs and strategies in advance. Accordingly, people often navigate through uncertainty by experimenting, or by depending on the information they get from other people to help them make economic decisions, assessing their relative payoffs all the while. With human decision-making, as with cells or ants or ions, strategies that produce relatively higher payoffs reproduce more frequently than those with relatively lower payoffs. For this reason, even human decision-making ends up displaying the complex patterns that characterize the other systems we have described, and can be analyzed in similar ways.

Some scholars in the legal academy (a very few as it turns out) have begun to use complexity economics, and the evolutionary dynamics that are embedded in complexity economics, to analyze law and legal regulation. In this chapter, we will explore the range of possibility that complexity economics might offer for the economic analysis of law. The chapter first discusses what complexity economics is, and the fields from which complexity economics draws. Second, it reviews with some specificity various methodological tools that complexity economists use. Third, the chapter explores three types of questions to which “complexity law and economics” might provide useful answers.

The main purpose here is to present the basic tools of complexity economics and then to suggest lines of inquiry for the economic analysis of law, so that the general reader can begin to explore the potential of what I am calling “complexity law and economics.” In so doing, we will review a range of literature from complexity economics and one or two examples of complexity law and economics, but will not systematically survey either genre of literature.

**What is Complexity Economics?**

First, is there such a thing as complexity economics? Indeed, does anything substantive unify the study of complex adaptive systems, the field from which complexity economics draws most heavily? Complexity scholars themselves see the field as a loose collection of methods and concepts drawn from a range of disciplines, unified around the study of dynamic interactions. As complexity

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5 Economist Brian Arthur tells the story that he came up with the term “complexity economics” on the spur of the moment, and only after an editor pressed him repeatedly on what to call this approach to economics.
At its most general, complexity economics sees the economy as a complex adaptive system, with interacting economic actors, much like an ant colony with interacting insects, or a meadow ecology, with species that interact to form complex patterns. To define terms a bit more formally, a complex system is a system that is composed of interacting elements in which the collective behavior produced from the interaction cannot be trivially predicted from or reduced to the behavior of the interacting elements.  

The basics of complexity economics are fairly simple. Economic actors interact at the local level primarily around price and information. Agents’ interactions are not random, but rather are structured by institutional networks. These networks—like banks, or firms, or free-trade agreement partnerships—mediate how, and how likely, actors are to interact with each other. 

Agents operate with incomplete information and with bounded rationality, using heuristics to navigate the decision-making that structures these interactions. Agents often learn from each other, by comparing payoffs to the majority, a trend-setter or the agents to whom they are most closely connected in their networks. 

Collectively, agents’ interactions create complex patterns of activity, buying, selling and trading, patterns of the sort that Adam Smith spoke of. Economic actors react adaptively to those patterns, and their reactions collectively modify the pattern in turn, to which they react again, in a mutually constitutive feedback loop. The patterns created from these recursive interactions emerge at the level of the collective. They can’t be reduced to, or predicted from, the rules of engagement at the level of micro-interaction. 

As is true of evolutionary dynamics more generally, the dynamics of the economy are frequently out of equilibrium but nevertheless display complex patterns—there are bubbles, crashes, runs, and periods of high volatility followed by periods of low volatility, for example. Patterns of activity are self-reinforcing in some instances, and show path-dependence via increasing returns. New markets, technologies and institutions are formed endogenously, and the economy displays a fair bit of novelty more generally.

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8 Id.

Most importantly, these interactions are dynamic, and evolve over time. The agents react locally to the overall patterns that they themselves have created. And in their reacting to that pattern, they again create patterns, to which the agents further react, and so on. In the context of this dynamic system, the key question that an economist might ask is, “How does this complex system unfold over time? How will the agents react to the patterns of the system? How will their reactions affect the pattern they have created?” Framed a little more formally, complexity economics asks how economic choices and strategies change endogenously with the patterns that these choices and strategies collectively create.

These are not the questions that neoclassical approaches to economics ask. Neoclassical economics asks a different sort of question, one that simplifies this complex set of questions. Generated in the absence of the kind of computational power that we now enjoy, neoclassical theories quite sensibly asks a shortcut sort of question that focused on consistency: “What agent behavior is consistent with, or in equilibrium with, the outcome or pattern that the agents’ behavior will cause?” So for example, general equilibrium theory asks what prices and quantities produced are consistent with the overall patterns of prices and quantities in the market. Classical game theory asks what best responses are consistent with the best responses of other players such that no one has the incentive to deviate.

These central questions focus economic inquiry on static equilibrium points that induce no further unfolding of patterns. Time and timing are not really relevant to much of the neoclassical analysis. More importantly, we remain in the dark about the way in which an equilibrium is formed or selected, and the dynamics of the economy leading up to that equilibrium formation. Equilibrium formation is for the most part disregarded by neoclassical analysis.

In contrast, dynamic models enable us to focus on much more—not just the dynamics leading to equilibrium formation but bubbles, crashes and other out-of-equilibrium structures. Non-equilibrium behavior arises because the economy involves processes that are at their base fundamentally unknowable in advance. So for example, what technological innovation will arise, how the public will receive it or how the government will regulate it cannot be predicted in advance. Moreover, this fundamentally unknowable technological change drives the development of other technologies that compliment earlier innovations—oil extraction technologies to allow gasoline production for the innovation of the automobile, for example.

10 See id.
11 Id. at 2
12 Id.
The usefulness of evolutionary dynamic models might best be understood if we think about the study of traffic. Realistic patterns of traffic movement demand a dynamic model that can describe patterns, novelty and endogenous change. Such models can help us to explain how a seemingly small and unpredictable event—shutting down a lane for a short time, the entry of a few more cars onto the freeway—can potentially trigger a dramatic phase shift from free-flowing traffic to massive gridlock on the freeways.  

Likewise, if we remove the restrictive filter that focuses on equilibrium, we are able to observe a much broader range of behavior that includes out-of-equilibrium dynamics, novelty and endogenous change. If we widen the lens to include experimentation, innovation, noise, boundedly rational behavior, non-equilibrium dynamics become a much more important focus of inquiry. We can focus on events like bubbles and crashes. We can investigate the complex interplay of move and counter-move, as those subject to legal regulation work to circumvent the regulation, and legal actors work to make them subject to regulation again.

A number of complexity economists have illustrated in a very simple way how broadening the lens of inquiry allows scholars to model economic behavior far more realistically. In the late 1980s and early 1990s, Brian Arthur, John Holland and other scholars at the Santa Fe Institute (“SFI”) set about constructing an artificial stock market with heterogeneous traders who adapted their expectations to the market (that those expectations had helped to collectively create.)

In their market, heterogeneous agents formed a market within the computer to trade a single stock, monitoring the price and submitting bids and offers. Importantly, these traders differed in the way they used available market information to predict future dividends. More specifically, traders used qualitative information plus data in a range of statistical ways that varied heterogeneously based on assumptions and error criteria.

Because their model included heterogeneous traders, Arthur and his team could not model the formation of expectations using conventional techniques. Ordinarily, conventional models assume that traders are homogenous, and so predicting others’ expectations is simple. All traders form their expectations in the exact same way, using data in the same way. The unfolding of such expectations collapses neatly into one deducible calculation.

With heterogeneity, however, the model’s traders were required to take into account others’ expectations of others’ expectations of others’ expectations (at least

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13 Id. at 9.

for $t = 3$), an infinite regress into a wide range of subjective expectations, a psychology of the market so to speak. To solve this problem, the team modeled traders more realistically, forming expectations via trial and error, learning and experience. Traders did not maximize; rather, they groped towards alternatives in the way that real traders do.

More specifically, Arthur’s team used a sort of genetic algorithm to model a Darwinian competition among market hypotheses. In this model, traders tested several different ways to use market information to predict dividends, and then kept the ones that predicted market movements well. Those that did poorly were dropped. And, importantly, occasionally agents would generate new alternative market hypotheses using the winning hypotheses as their foundation for experimentation.

The SFI model and its follow-ups produced quite fascinating results. When the rate at which new alternative market hypotheses (the “exploration rate”) was low, the market converged rapidly to a rational expectations value adjusted for risk. Trading volume was low and bubbles and crashes did not emerge. But when the exploration rate was high, the market demonstrated several key features that more closely resembled real market behavior.

In particular, the market displayed bubbles and crashes, at all scales, arising spontaneously as agents reacted to market patterns. Returns displayed long tails (meaning that they showed results with probabilities that were quite far from the median or mean), and prices looked quite realistic (non-Gaussian) in their distribution. Periods of high volatility were followed randomly by periods of quiescence. Small events caused cascades of change ricocheting through the markets, and the market displayed volatility clustering. These properties are empirically observable across a wide range of markets.

The heterogeneity of market hypotheses, and the feedback effects of agent expectations appeared to be responsible for much of the artificial markets’ real-world behavior. As was true in the real world, agents did not all use market information in the same way, and based their market hypotheses on trial and error. And, as was true in the real world, history seemed to matter. Small events could endogenously produce self-reinforcing bubbles and crashes, owing to the way in which market traders’ decisions built on earlier market patterns.

Arthur’s work illustrates the flexibility that complexity economics can offer. Such models can incorporate more realistic assumptions about heterogeneous processing of information. And correspondingly, the models produce system dynamics that more closely resemble the volatile moods of the market, moving from periods of

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15 See id. See also Arthur, Complexity Economics, supra note x.
relative quiescence to periods of greater volatility. The next section discusses with more specificity some of the tools that complexity economics uses.

**Complexity Economics Methods**

As is true for complex systems theory, complexity economics is a fundamentally interdisciplinary project, drawing together a loose core of methods from several disciplines that focus on describing systems of interacting agents that adapt over time. As such, complexity economics can be said to draw from four or five key theoretical areas.

First, complexity economists draw on the study of dynamics, and in particular, the dynamics of adaptive systems. To study these dynamics, scholars rely typically either on analytical mathematical techniques to derive solutions to equations, or numerical simulations of a dynamic system to observe the dynamics more directly. In terms of analytical techniques, many mathematical models of the dynamics of the economy are equation-based, and rely on differential equations and their solutions to describe the dynamic patterns that interacting agents create. For example, we can use an equation-based model that relies on differential equations to describe the diffusion dynamics of macro-economic forecasts by a small set of agents across a population of economic decision makers. In terms of numerical simulations, complexity theorists also make use of agent-based models, which numerically simulate the interactions of heterogeneous agents. In an agent-based model, dynamic equations are replaced by rules of interaction, and the dynamics of the system are explored over a range of parameters. These two kinds of evolutionary dynamics models will be discussed more fully in the following section.

Complexity economics scholars also use theoretical tools from the study of information processing. Much work draws inspiration from Herbert Simon’s study of the way in which people operate under conditions of limited information or high uncertainty. These models assume that people navigate in the face of uncertainty by making use of effective heuristics—rough and ready rules of thumb—to operate in complex or novel environments. While neoclassical economics has focused on the way in which these heuristics are responsible for irrational decision-making, complexity economists focus on the upside of such heuristics, and the way in which such decision-making can quickly and efficiently push a system into productive regions of search space.

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16 Christopher Carroll, The Epidemiology of Macroeconomic Expectations, in The Economy as Complex Evolving System III (Blume and Durlauf eds. 2005).

For example, some evolutionary game theorists in economics use imitation and learning models that assume that under conditions of uncertainty, market actors will compare their payoffs with those of others around them—a market leader, their neighbors, the majority, a teacher or mentor—and will switch to the other’s strategy with some probability. The artificial stock market is an example of another type of model in which the market actor herself tests competing hypotheses using genetic algorithms that borrow heavily from computer science and artificial intelligence. There are many models of social learning and imitation, and these models draw heavily from sociological and social psychological empirical study about the way economic actors make decisions in a social context.

Political economy and economic history play important roles in complexity economics as well. As is true for many evolutionary processes, economic outcomes can be path-dependent; small events in history can strongly influence the evolutionary trajectory of technology markets, for example. This is because many economic processes are self-reinforcing over time. Small events can trigger cascades of change that propagate through a system quickly and with increasing speed, much like an infection sweeping through a population.

Complexity economists also draw insight from network and graph theory. In a complex economy, market actors interact in ways that are structured by sophisticated institutional and structural networks. Such networks make it more likely that some market actors interact than others, and that people’s choices are a function of those connected to them in economic networks. For example, scholars have modeled the spread of financial collapse as propagated along networks of debt cross-holding in key European countries and the network structure of favor-exchanges in villages in India.

As will be discussed later in the chapter, economic outcomes produced from interaction along these networks often show unexpected dynamics.

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19 For an excellent explanation of the connection between economic history and complexity, see Douglas Puffert, Path Dependence, Network Form and Technological Change, in History Matters: Economic Growth, Technology and Population (Sundstrom et al eds. 2003).

20 Matthew Elliot, Benjamin Golub and Matthew Jackson, Financial Networks and Contagion (2013).

economists call these effects “emergent”—that is, they cannot be predicted from the micro-interactions at individual nodes and links of the network.

Finally, complexity economists draw a great deal from evolutionary theory. Neoclassical theorists have considered, and dismissed, evolutionary theories of institutional development. Complexity economics has revived the general notion that in a complex economy, patterns of activity evolve over time, responding to selective pressures created by various endogenous features of the economic environment. Tools from evolutionary dynamics, computational biology and evolutionary game theory (which will be discussed in more detail below) are quite popular among complexity economists.22

**Evolutionary Game Theory Approaches**

Unlike conventional game theory, evolutionary game theory studies the behavior of large populations of agents who interact over time. Introduced as a way to deal with large populations in biology, John Maynard Smith and George Price developed evolutionary game theory models of animal behavior in the 1970s. In these models, payoffs are synonymous with reproductive fitness. Higher-fitness strategies spread in the population, and less successful strategies diminish or go extinct.

Evolutionary game theory differs from classical game theory in two important ways.23 First, evolutionary game theory deals with populations of individuals instead of a small number of players. Second, evolutionary game theory makes room for the evolutionary dynamics of strategic behavior. Evolutionary game dynamics can model both biological and cultural evolution, in which strategies spread by way of imitation and learning. These models can incorporate more realistic bounded rationality, in which myopic individual agents decide whether to switch strategies on the basis of some comparison of their payoffs with a small number of other agents.

In this dynamic vein, evolutionary dynamics include a range of rules for social learning over time. One of the most commonly studied dynamics is pair-wise comparison, in which an agent switches to the strategy of a randomly selected role model with some probability proportional to the difference in fitness between the agent and the model.24 Although critics complain that the selection of the updating mechanism is arbitrary, it is important to remember that updating mechanisms can

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23 See id.

be roughly grouped into categories—for example, imitative dynamics—and that similar mechanisms generate qualitatively similar aggregate behavior.\textsuperscript{25}

As noted earlier, EGT models come in two general categories, analytical and agent-based, each with useful features to scholars interested in population dynamics. If the population is large then the deterministic dynamics of the strategies in the population can be approximated by analytical solutions to an ordinary differential equation. For example, Taylor and Jonker introduced the replicator dynamic equation, in which the frequency of a strategy increases when its fitness exceeds the average fitness of the population.\textsuperscript{26} Extending this basic population game to allow nonlinear payoffs extends the model's application without making the analysis too difficult. Scholars have extended replicator dynamics in a range of ways, to incorporate varying degrees of bounded rationality in the analysis.

Deterministic dynamics can be assessed with pretty simple analytical techniques. In particular, using basic principles of linear algebra and other mathematical tools, analysts can determine the stability of rest points by linearizing around the equilibria. Rest points are asymptotically stable if after a slight perturbation, the dynamic converges back to the rest point. Unstable rest points move away from the rest point, and neutrally stable points leave the perturbed system at a point near the original rest point. In many cases, the stability of the rest point can be determined by assessing whether the eigenvalue of the Jacobian matrix of the payoff vector field is negative.\textsuperscript{27}

So for example, in a dynamic game of hawk and dove, in which a population of agents chooses whether to play hawk or dove, the change in frequencies of these strategies can be described using two differential equations, and these equations can be solved analytically to describe the rest points (equilibria) that these frequency changes converge to, and their stability as well.

In contrast, agent-based models allow us to perform numerical simulations across a range of parameters, for those systems in which it is too difficult to describe the behavior analytically. Agent-based models focus on the micro-mechanisms among individuals to derive larger patterns of behavior in a bottom-up way. In agent-based simulation models, scholars describe the underlying mechanisms of interaction with simple fixed rules rather than via equations.\textsuperscript{28} Agent behavior is encoded into

\textsuperscript{25} Id.

\textsuperscript{26} P.D Taylor and L. Jonker Evolutionarily Stable Strategies and Game Dynamics, 40 Mathematical Biosciences 145 (1978).

\textsuperscript{27} Id.

\textsuperscript{28} Scott Page, Agent Based Models in New Palgrave Dictionary of Economics (2d ed. 2008).
computer programs by way of these rules and the dynamics of their interaction are simulated numerically.

For example, an agent-based model of hawk and dove could program the rules of interaction between hawks and doves. We can then explore the dynamics of the game by inputting a range of parameters—payoffs, the precise mechanisms by which strategies spread, among others—and observing the results. In particular, we can observe the change in frequency of strategy in a population over time.

The dynamic public goods game is a particularly useful game to model for legal scholars. Both types of evolutionary game theory—analytical and numerical simulation—easily model public goods games, which represent a generalization of the classic Prisoner’s Dilemma, only for interaction groups of large size. In a public goods game, agents have the opportunity to invest at cost c into a common pool or to defect and contribute nothing. The common pool total is then multiplied by some factor and afterwards, divided evenly among all participants whether or not they contributed to the pool.

Public goods games are very useful for studying the co-evolutionary dynamics of legal punishment, not just by individual agents but also by centralized agents like “the State.” Most public goods punishment games punish an agent by imposing a fine, at some cost to the punisher. The impact of the fine changes the payoff structure for the choice of whether to contribute, and the cost of punishment shapes the choice of whether or not to punish.

Complexity economics has its critics, to be sure. More than a few people have objected in particular to agent-based models, in which scholars simulate the dynamics of interacting agents, rather than deploying analytical tools. Here, the critics argue that replacing analytical solutions with numerical simulations that are worked out computationally makes the method far less rigorous, and far more subject to confusion about what drives outcomes. But practitioners point out that equation-based methods often require assumptions and abstractions that are highly unrealistic, to make the method tractable.29 Agent-based modeling, and complexity more generally, supplements such tools, with an eye towards improving both the realism of the assumptions and of the outcomes.

What is Complexity Law and Economics?

So what might complexity law and economics look like, and how might it differ from neoclassical and post-neoclassical law and economics? Let us begin with a deliberately oversimplified account of modern law and economics as a point of departure. Contemporary law and economics studies legal rules and institutions using economic theory and econometric methods. The engine that powers this investigation is the insight that legal sanctions or rewards affect behavior in much the same way that prices affect behavior in economic theory—by affecting the payoffs of behavioral strategies.

Seen through the lens of economic theory, legal rules work by changing the payoff structure of alternative decisions. Legal fines or criminal penalties lower the payoffs of certain choices. Legal subsidies or rewards raise the payoffs of other strategies. And in a rational expectations model, people respond to those incentives rationally, just as they respond to price. Normatively, the discipline focuses (some might say obsessively) on generating legal rules that produce an "efficient" equilibrium outcome.

To use a common example, the neoclassical incarnation of law and economics asks a set of questions about the relationship between legal punishment and criminal behavior in equilibrium. How does a change in the probability of punishment affect the equilibrium level of crime? Evolutionary game theory asks a different set of questions. Assume that would-be criminals do not know in advance what the probability of punishment is, nor do they know the magnitude of the penalty or the benefits to committing a crime; rather, they decide by comparing their payoffs to other people’s payoffs, and switch to committing a crime with some probability if they observe that other people’s payoffs are higher. How would a change in the probability of punishment affect the dynamic spread or containment of crime? Under what conditions could lowering the punishment probability and increasing the magnitude of the penalty produce a less-costly way to contain crime? If the system reached an equilibrium level of crime, would it be a stable equilibrium or unstable?

More generally, neoclassical approaches assume, as a useful shortcut to investigation, that human behavior reaches some equilibrium, and that changing legal rules shift these equilibria by changing payoffs. But complexity theory might


31 Daria Roithmayr, Shmuel Leshem and Moshe Hoffman investigate this question in forthcoming work. Roithmayr, Leshem and Hoffman, An Evolutionary Economic Analysis of Law and Punishment.
allow us to ask an additional set of questions that could perhaps be more useful for some purposes.

In particular, the evolutionary dynamics of law might help us to ask and answer three categories of questions:

**First**, how does a change in legal rules affect the complex unfolding patterns of interactions (and not just the timeless equilibrium interactions) among people? For example, how would a change in property rules from collective ownership to individual ownership affect the dynamics of cooperation, individual investment and punishment? As the following discussion will elaborate, recent work shows unexpected out-of-equilibrium results.

**Second**, how does law itself structure the complex unfolding patterns of interaction? Law almost always shapes the world that it regulates, and in so doing, structures the networks and institutions that mediate interaction. As the next section demonstrates, the overall flow of traffic on a telecommunication network emerges in unpredictable ways from the interaction of individual nodes and links. Regulatory requirements that give competitors access to those individual nodes and links can have unexpectedly negative effects by literally restructuring the networks.

**Third**, and finally, how does law evolve, endogenously and in tandem, with the behavior it regulates? Just as law shapes and structures the unfolding patterns of regulated behavior, so too is law shaped by those patterns. For example, Congress adaptively revises its tax rules in response to innovative tax shelters created by taxpayers who in turn are seeking to adapt to current legal rules. Payday lenders adapt their strategies to circumvent state regulation, and the federal government must then step in to try to regulate. What is the nature of this co-evolutionary relationship? How do the relative rates of evolution by law and society affect rates of compliance? The following discussion considers each of these three categories of inquiry in turn.

1. **How does a change in law affect the complex unfolding patterns of behavior that law regulates?**

Law works largely by regulating patterns of behavior. For example, traffic (like the economy) spends much of its time out of equilibrium. Micro-interactions among cars produce regular and complex patterns—for example, as a few more cars enter the freeway, the traffic exhibits a very dramatic phase-shift between free-moving traffic and jammed traffic. Legal rules about speed, which are designed to regulate the flow of traffic, can affect the complex patterns of traffic in unexpected ways that can’t be predicted from the rule in advance.

Complexity law and economics might allow us to investigate the relationship between legal rules and those more complex patterns. How would a change in intellectual property rules affect the out-of-equilibrium dynamics of radical
innovation on the one hand and exploitation of current ideas on the other? How would a change in private property rights affect the dynamics of cooperation among farmers or hunters?

In a recent paper, Sam Bowles and Jung Kyoo Choi have investigated the evolutionary relationship between property rights and the non-equilibrium unfolding of the transition between hunter-gathering and farming around 12,000 years ago. Bowles and Choi argue that property rights co-evolved with new farming technology. In their story, the institution of property rights and the new farming technology were unable to advance on its own, but each made the other possible. For the moment, let us focus very narrowly on the part of their paper that investigates the relationship between norms of private property rights and the non-equilibrium dynamics of regulated behavior.

Bowles and Choi use a computational agent-based model that numerically simulates agent interactions according to simple rules of engagement. In their model, agents first decide whether to forage or farm; farming requires a prior investment and because farm plots are well defined, it pays more to stake a claim to farm plots than to stake and defend a claim to a deer carcass for the hunter.

Agents then decide whether to share the fruits of their labor and afterward, whether to collectively punish those who do not share. Agents can choose one of three strategies: to share their profits (“Sharers”), to keep their profits for themselves (“Bourgeois”) and to both share and collectively punish those who don’t share (“Civics.”). Importantly, after each round of play, agents compare payoffs with another randomly selected agent in the group, and switch (probabilistically) to their partner’s strategy if the partner’s payoffs are higher. Occasionally, groups compete against each other, and the groups with higher payoffs survive to spread its strategy.

The model’s results are fascinating, and illustrate well the effect of legal rules on unfolding patterns of cooperation. Let us look at the results for the hunter economy: in the model, the dynamics converge to two stationary points, one stable and the other “neutrally stable,” meaning neither converging nor unstable.

More specifically, a population of hunters will come to an equilibrium that consists of a mix of Sharers and Bourgeois. Civic hunters cannot take over the population because as “the enforcers,” they must pay the cost of punishing Bourgeois hunters,

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32 Bowles and Choi refer to the grabbers as “bourgeois” and sharer-punishers as “civics.” I will use the term grabbers and sharer-punishers for ease of exposition.

33 Interestingly, the authors tried to approximate the parameters and payoffs to accord with available data on the benefits of hunting and farming, and amazingly, the weather conditions during the relevant periods. Id.
which none of the other strategies must pay. Bourgeois and Sharer hunters co-exist in a sort of mutual détente. Bourgeois hunters cannot take over Sharer hunters because the former face a significant liability when there are too many in the population—whenever two Bourgeois hunters meet, they fight over the ownership of some foraged good, and it does not pay well to defend this good because it is hard to define ownership as easily as one can with a plot of land. At the same time, Sharer hunters cannot take over Bourgeois hunters because the Sharers must give up some of their goods to the others.

More interestingly, the results demonstrate that a second stationary state is also stable, but only neutrally so. In addition to the mix of Bourgeois and Sharers, the population might also come to equilibrium with a population of all Civic hunters. This homogenous groups does well because the Civics collectively will punish any Bourgeois mutant who arises to threaten the group. In a homogenous Civic population, because Bourgeois mutants only occasionally arise, Civics rarely bear the cost of such punishment and so they enjoy no competitive advantage over Sharers.

But this group is vulnerable to drift. Because there is little comparative advantage among them, Sharers and Civics become virtually equivalent and as a matter of chance, the system will move (or “drift”) towards more Sharers than Civics. The mix of Sharers and Civics will remain neutrally stable so long as enough Civics are available to punish the occasional Bourgeois effectively. But once there are too few Civics to punish, the mix then becomes vulnerable to Bourgeois agents. Bourgeois hunters now replace Civic hunters, and the system moves toward the first stable point described above.

The point to make here is that conventional techniques miss both the neutrally stable point, and more importantly the dynamics of the system. Bowles and Choi’s agent-based model can usefully describe the way that different legal norms generate different stable points, and the way in which the system behaves when it is out of equilibrium. Descriptions of the neutral drift from a mix of groups to a critical threshold are possible only in models that allow for dynamic change. More generally, agent-based models allow the scholar to conduct experiments to predict the effect of legal change on regulated behavior (and as we will see in later discussion, on subsequent legal change.)

2. **How does law structure the dynamics of regulated behavior?**

In picturing the social interactions that law regulates as part of a complex adaptive system, complexity law and economics might also help us to understand the way in which law actually constructs that system. In the most foundational sense, law generates the rules that govern local interaction among agents. For example, law often requires the disclosure of certain kinds of information between a buyer and seller, where the parties might not ordinarily be inclined to share such information.
More importantly, legal rules often determine the probabilities of who interacts with whom in a complex adaptive system. Interaction among people is far from random. Institutional and social networks are structures that make interaction among certain people who are linked by network connections far more likely than people who are not linked. Law structures those network connections. Legal rules often work by requiring actors to interact, and by structuring the nature of those interactions.

But as evolutionary network theory makes clear, we cannot predict the effect of changes to those network-constructing legal rules. Because a change to the network often produces cascades or ripples that affect the entire network configuration, the overall pattern of network flow cannot be reduced to or predicted from the behavior of network elements in isolation. Small changes in the individual elements can produce unexpected larger-scale effects—bottlenecks, congestion, traffic jams and so on.

An example from telecommunications is most instructive in this regard. In an effort to promote open access and competition, the 1996 Telecommunications Act required that an incumbent network operator’s emerging competitors have access to the network, on terms that the competitor was largely free to decide. Using complex systems theory, legal scholars Daniel Spulber and Christopher Yoo have modeled the way in which competitor access, even to links in the network that appear to have sufficient capacity, can unexpectedly reduce the capacity of the entire network, and render pricing schemes incoherent.

In particular, required competitor access can diminish network performance by affecting a set of critical links through which all traffic must flow, like the bridges to Manhattan across which everyone must drive to get off the island. A network’s point of vulnerability is its set of smallest bridges or weakest links through which all traffic must flow. If a competitor accesses capacity on a link that is already one of the weakest links, or if access on a link actually now pushes that link into that weakest link set, then the overall performance of the network may be reduced.

Graph theory shows us how changing capacity at any one of the links in the network can have unexpected effects on the dynamics of traffic flow in the entire network by affecting that set of weakest links. Compounding the difficulty, incumbent network operators are not allowed to decide where on the network competitors gain access; the law specifies that the competitors can choose their point of access at any technically feasible point.

More concretely, the effects of legally requiring competitor access on overall network performance makes pricing wholly disconnected from the effect that access

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has on network performance. For some links, at some particular point in time, competitor access will diminish performance. For other links, or the same link at other times, competitor access will have no effect or actually improve performance. Pricing access by looking only at the link to which the competitor gains access bears no relation to the actual costs of that access. Competitor access in effect changes the traffic patterns through the network.

The authors argue that just as transportation policymakers must take into account the dynamic effects of adding a new on or off-ramp to traffic patterns on the entire network, so should policymakers consider the effect of permitting competitor access on the entire network. More generally, complexity law and economics models of the sort that Spulber and Yoo deploy can help us to understand the way in which law affects outcomes by structuring the network of interaction among players.

3. How does law evolve, endogenously and in tandem, with the behavior it regulates?

Perhaps most interestingly, complexity law and economics might give us the tools to investigate how law evolves in tandem (“co-evolves”) with the behavior it regulates. Biologists study the co-evolutionary relationship among species that evolve in response to each other. For example, scientists recently have discovered that a species of poisonous newt has evolved in some localities very high levels of poison in response to selective pressure created by a predator snake. And the same species of snake has evolved in those same localities almost perfect immunity to newt poison.

Co-evolutionary models specify two sources of selective pressure that drive the evolutionary arms race dynamics. First, between the interacting species, for example those in predator-prey interactions, each species creates selective pressure that influences the evolution of the other. For example, the snake that develops greater immunity will eat more newts, and this predatory strategy will be reproduced more frequently in subsequent generations. At the same time, more poisonous newts will evade capture, and their reproductive strategy will appear with greater frequency within the species. Immunity and poison give the species comparative advantage against each other.

35 *Id.* at 1710.

36 These co-evolutionary dynamics are not inevitable. Indeed, the research demonstrates that for some localities, traits between the species are so mismatched that either the snakes or the newts win decisively, and the arms race never gets off the ground. C. Hanifin, E. D. Brodie, Jr. and E. D. Brodie III. Phenotypic mismatches reveal escale from arms-race coevolution, 6 PLoS Biology 471 (2008).
Second, within the species, individuals compete for limited resources and that internal competition creates selective pressure. For example, immunity and poison give individual newts and snakes comparative reproductive advantage against each other.

Co-evolutionary models might usefully describe the relationship between legal rules and the behavior those rules regulate. At the level of interaction between legal rules and regulated behavior, legal rules can evolve in response to adaptive innovation in regulated behavior. For example, in tax policy or financial regulation, taxpayers and financial players often innovate to produce the next new “legal” tax shelter or new complex financial product with astonishing speed.

At the level of competition within legal rules and within regulated behavior, lawmakers will compete with each other as will those who are regulated compete with each other. For example, in the area of tort liability and corporate governance regulation, states frequently compete with each other to attract business and tax revenue. A co-evolutionary model can usefully describe the complex dynamics that integrate both the interaction between legal rules and regulated behavior on the one hand and the competition among legal rules on the other (and competition among the regulated as well.)

Evolutionary game theory in particular can be quite useful in developing a framework with which to understand the co-evolution of law and the behavior law regulates. More specifically, such models can help us focus on how to structure law so as to improve citizen compliance.

Recently, Roithmayr, Rand and Isakov have begun to investigate the rate of evolution for legal rules, and more specifically, the effect that varying that rate might have on citizen compliance. Typically, legal rules evolve or “update” more slowly than the behavior that the rules regulate. Recent research in biology reveals that in some circumstances, a more slowly-evolving species can obtain a comparative advantage over a cooperating species by “committing” to a strategy. At the same time, in more antagonistic relationships, evolving more quickly produces a comparative advantage by “outrunning” the competing species.

The Roithmayr team’s model investigates the rate at which legal rules evolve relative to the evolution of citizen behavior—so for example, the rate at which tax law evolves relative to the rate at which citizens generate new tax shelters. The authors generate an agent-based model in which citizens play a public goods game with centralized State punishment. Citizens decide how much to contribute to the State’s tax coffers. Their contributions are multiplied and then divided pro rata among all citizens, regardless of how much the citizen contributes (which creates

37 Daria Roithmayr, David Rand and Alex Isakov, The Rate of Legal Change (unpublished manuscript, on file with author.)
the incentive to free-ride, in the absence of punishment.) States punish the non-contributors at varying levels.

Citizens compete with each other and learn from each other. States also compete with and learn from each other. States face budget constraints—the State’s ability to punish depended on how effectively citizens contributed to the public good. In turn, citizen’s level of cooperation in contributing depends on the State’s ability to punish effectively.

Numerical simulations reveal that States can maximize citizen cooperation when the rate at which their legal rules evolve hits a critical rate. This critical “update rate” must be sufficiently slow relative to citizens’ update rates that the citizens are forced to adapt to their State’s legal rule. By adopting a relatively slow rate, states effectively tie their hands and commit to costly punishment. Essentially, the state with a relatively slower update rate has no capacity to reduce the level at which it is punishing, even when large groups of agents are defecting. So long as the state adapts slowly enough for the citizens to “learn” to respond to punishment, the state’s punishment will be effective.

But States cannot be too slow. The model also shows that when States face budget constraints that condition their enforcement revenue on citizen contribution, the State becomes vulnerable to citizen defection. Accordingly, the critical rate of legal evolution must also be sufficiently fast that states can adapt in response to widespread and contagious defection. In the face of limited resources to prosecute defectors, States that are too slow to adapt cannot react to widespread defection by re-allocating resources in a cost-effective way. States that are too slow to react cannot reduce punishment levels to a level that the State can afford as a way of regaining the ability to punish.

This co-evolutionary model constitutes an initial step towards developing a more robust and useful co-evolutionary model of legal rules and citizen strategic behavior.

**Conclusion**

Complexity law and economics models offer a range of flexible tools that scholars interested in a dynamic economic analysis of law can use. Complexity law and economics models make room for non-linear dynamics, with endogenously generated events, and systems that spend large amounts of time out of equilibrium. The dynamic approach intrinsic to such a model is particularly apt to describe heterogeneous behavior, in both lawmaking and the decision-making of those who are regulated by law.

The agents in complexity law and economics models can display more realistic limits on their rationality. They can use heuristics to navigate their way through uncertainty. They can learn from each other and from experience, over time. Most notably, the complexity law and economics framework allows us to see agents react
to the patterns they create, and then to create new patterns or recreate the old ones. For legal scholars, who work at a scale in which history and time matter, the ability to describe the dynamics of law and society is quite useful.

Complexity law and economics models offer scholars the ability to more usefully describe the co-evolutionary relationship between law and regulated behavior. Such models can potentially incorporate both the way in which legal rules and regulatory regimes compete with each other and affect citizen behavior, which in turn affects the creation of legal rules themselves.

Finally, complexity law and economics models might more easily make room for innovation, as the SFI artificial stock market model demonstrates. Innovation is not imposed from the outside but endogenously generated. New legal rules generate new unmet legal needs (“how do we price competitor entry into a network?”) that then give rise to new legal technologies to meet those needs, and so on. Innovators tweak old legal rules and recombine their parts with other parts of other rules (using a disability framework to analyze discrimination, for example.)

Complexity law and economics methods might also offer legal scholars the opportunity to draw useful insights not just from their colleagues in economics and the social sciences but also from biology, physics and other areas that to date have offered less to the legal scholar. Legal scholars in general, and law and economics scholars in particular, have much to gain from such collaboration.